

**Definition 0.1.** In order to define the [concept](#) of *functor category*, let us consider for any two [categories](#)  $\mathcal{A}$  and  $\mathcal{A}'$ , the class

$$\mathbf{M} = [\mathcal{A}, \mathcal{A}']$$

of all covariant functors from  $\mathcal{A}$  to  $\mathcal{A}'$ . For any two such functors  $F, K \in [\mathcal{A}, \mathcal{A}']$ ,  $F : \mathcal{A} \rightarrow \mathcal{A}'$  and  $K : \mathcal{A} \rightarrow \mathcal{A}'$ , let us denote the class of all [natural transformations](#) from  $F$  to  $K$  by  $[F, K]$ . In the particular case when  $[F, K]$  is a set one can still define for a [small category](#)  $\mathcal{A}$ , the set  $Hom_{\mathbf{M}}(F, K)$ . Thus, cf. p. 62 in [1], when  $\mathcal{A}$  is a *small* category the ‘class’  $[F, K]$  of natural transformations from  $F$  to  $K$  may be viewed as a subclass of the cartesian product  $\prod_{A \in \mathcal{A}} [F(A), K(A)]$ , and because the latter is a *set* so is  $[F, K]$  as well. Therefore, with the categorical law of [composition](#) of natural transformations of functors, and for  $\mathcal{A}$  being small,  $\mathbf{M} = [\mathcal{A}, \mathcal{A}']$  *satisfies the conditions for the definition of a category*, and it is in fact a *functor category*.

**Remark:** In the general case when  $\mathcal{A}$  is *not small*, the proper class  $\mathbf{M} = [\mathcal{A}, \mathcal{A}']$  may be endowed with the structure of a [supercategory](#) (defined as any formal interpretation of [ETAS](#)) with the usual categorical [composition law](#) for natural transformations of functors. Similarly, one can construct a *meta-category* defined as the *supercategory of all functor categories*.

## References

- [1] Mitchell, B.: 1965, *Theory of Categories*, Academic Press: London.
- [2] Refs. [15], [17], [18] and [288] in the Bibliography of Category Theory and Algebraic Topology. Categories, Functors and Automata Theory: A Novel Approach to Quantum Automata through Algebraic-Topological Quantum Computations., *Proceed. 4th Intl. Congress LMPS*, P. Suppes, Editor (August-Sept. 1971).